CGWAS Problem Set - EM Counterparts (Brian Metzger)

1 Kilonova

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A NS-NS binary merger ejects a mass $M_{\rm ej} = 10^{-2} M_{\odot}$ of r-process nuclei with a mean velocity of $v_{\rm ej} = 0.1$ c. In this problem you will calculate the kilonova signal using the spherical cow model presented in lecture.

As we discussed, the thermal energy E in the one-zone approximation evolves according to

$$\frac{dE}{dt} = -\frac{E}{t} - \frac{E}{t_{\rm diff}} + \dot{Q}_{\rm ext}, \tag{1}$$

as a function of time t, where

$$t_{\rm diff} = \frac{3\kappa M_{\rm ej}}{4\pi c v_{\rm ej} t} \tag{2}$$

is the characteristic photon diffusion timescale, κ is the opacity, and

$$\dot{Q}_{\rm ext} \approx 10^{10}\,{\rm erg\,g^{-1}\,s^{-1}} \times M_{\rm ej} \times \left(\frac{t}{\rm day}\right)^{-1.2} \tag{3}$$

is the heating rate due to radioactive decay of the r-process nuclei.

1. Numerically solve equation (1) for $E(t)$ for $t > t_0$. You may find section III.9 of this reference useful for how to numerically solve ODEs.[1] You may assume as an initial condition that the thermal energy of the matter is comparable to its kinetic energy, $E(t_0) = M_{\rm ej} v_{\rm ej}^2/2$ at the time of ejection $t_0 = R_0/v_{\rm ej}$, where $R_0 \approx 10^7$ cm is the radius at which matter is ejected from the merger (your results should not be sensitive to $E(t_0)$ as long as t_0 is sufficiently small). Assume an opacity of $\kappa = 10$ cm^2 g^{-1} appropriate for lanthanide/actinide-rich ejecta.

2. Plot or sketch the radiated power (light curve), $L_{\rm rad}(t) = E/t_{\rm diff}$ versus time. For comparison also plot the radioactive heating rate $\dot{Q}_{\rm ext}(t)$. At what time does the light curve peak? How can this be understood on simple theoretical grounds? If $\dot{Q}_{\rm ext}(t) \ll L_{\rm rad}$ at any time, where is the extra energy from radioactive decay going?

[1] http://www.tapir.caltech.edu/~cott/ay190.pdf

3. Calculate and plot the emission temperature versus time. You may assume blackbody emission,

$$T = \left(\frac{L_{\rm rad}}{4\pi\sigma R_{\rm ej}^2} \right)^{1/4}, \tag{4}$$

where $R_{\rm ej} = v_{\rm ej} t$ is the ejecta radius. What is the temperature at peak light? At what wavelength will the signal peak?

2 Short GRB

The merger of a NS-BH binary results in the tidal disruption of the NS, forming a torus of mass $M_{\rm t} = 0.1 M_\odot$, characteristic radius $R_{\rm t} = 40$ km, and characteristic radial thickness $\sim R_{\rm t}$. The vertical scale-height of the torus, H, is ten percent of its radius, i.e. $H = 0.1 R_{\rm t}$ (this is set by the balance between turbulent heating and neutrino cooling). The mass of the BH is $M_\bullet = 8 M_\odot$. You may find this reference useful for the basic physics of accretion disks.[2] *Note: all of the problems below are meant to be order-of-magnitude estimates!*

1. Write down an approximate expression for the midplane density of the torus, $\rho_{\rm t}$, using just the variables given above. Your answer need only be accurate to a factor of a few.

2. Write down an approximate expression for the midplane pressure of the torus, $P_{\rm t}$. Neglect the self-gravity of the torus. Your answer need only be accurate to a factor of a few.

3. Write down an approximate expression for the isothermal sound speed in the midplane of the torus, $c_{\rm s}$ in terms of just $M_\bullet$, H, $R_{\rm t}$.

4. Estimate the characteristic timescale for a sizable fraction (e.g. half) of the torus to accrete onto the BH. You may assume that magnetic turbulence in the disk produces an effective viscosity $\nu = \alpha c_{\rm s} H$, where $\alpha = 0.01$ (similar to that measured in shearing-box simulations of the magneto-rotational instability). Express your result in seconds.

5. What is the characteristic accretion rate, $\dot{M}_{\rm t}$, of the torus in solar masses per second?

6. Assume that accretion powers the formation of a relativistic jet. Calculate the power of the jet $L_{\rm j} = \epsilon_{\rm j} \dot{M}_{\rm t} c^2$ for an assumed jet efficiency of $\epsilon_{\rm j} = 0.01$.

7. Write down an approximate expression for the total angular momentum of the torus, $J_{\rm t}$.

8. Assuming that $J_{\rm t}$ is conserved as matter accretes, how does the disk radius $R_{\rm t}$ evolve as a function of the decreasing torus mass, $M_{\rm t}$?

[2] http://www.astronomy.ohio-state.edu/~ryden/ast825/ch9.pdf