CGWAS Problem Set - EM Counterparts (Brian Metzger) - SOLUTIONS

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1 Kilonova

1. Solution for E(t).

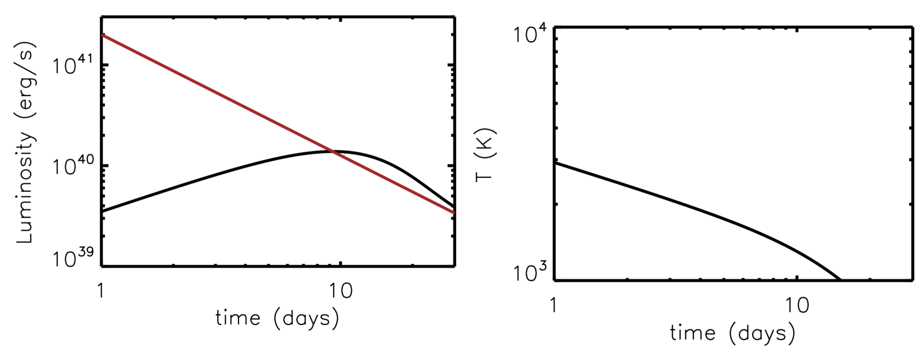
2. The light curve and temperature are shown in Fig. 1 below. The light curve peaks at about 10 days at a luminosity of $L_{\rm rad}(t_{\rm pk}) \approx 10^{40}$ erg s^{-1} . This can alternatively be derived from the simple analytic estimate given in lecture, whereby we equate the diffusion time (eq. 2) and the expansion time t to obtain

$$t_{\rm pk} = \left(\frac{3\kappa M_{\rm ej}}{4\pi c v_{\rm ej}}\right)^{1/2} \approx 8.4\,{\rm day} \tag{1}$$

At times $t \gtrsim t_{\rm peak}$ the luminosity tracks the rate of energy input from radioactive decay, such that

$$L_{\rm pk} \approx \dot{Q}_{\rm ext}(t_{\rm pk}) \approx 1.5 \times 10^{40}\,{\rm erg\,s^{-1}} \tag{2}$$

At times $t \ll t_{\rm peak}$ we have $L_{\rm rad} \ll \dot{Q}_{\rm ext}$. The fraction of $\dot{Q}_{\rm ext}$ not radiated at early times is going into PdV work on the ejecta (the first term in eq. 1). Technically the speed of the ejecta should be increasing, but the energy released by radioactive decay is so much smaller than the initial kinetic energy of the ejecta that this increase in velocity is generally neglected.



(a) Luminosity $L_{\rm rad} = E/t_{\rm diff}$ (black) and radioactive heating rate $\dot{Q}_{\rm ext}$ (brown).

(b) Temperature as a function of time.

3. The peak temperature is about $T_{\rm pk} \approx 1200$ K, corresponding to a spectral peak at $\nu = 3kT_{\rm pk}/h \approx 8 \times 10^{13}$ Hz, or $\lambda \approx 4\mu$ m, i.e. in the near-infrared. Actual calculations (e.g. Kasen, Fernandez, Metzger 2015) show that the spectrum peaks somewhat bluer than this, since the assumption we have made of a gray opacity is not entirely accurate.

2 Short GRB

1. The torus is spread over a characteristic radius $R_{\rm t}$ and its vertical thickness is $2H$ so the midplane density is approximately given by

$$\rho_t \sim \frac{M_{\rm t}}{2\pi R_{\rm t}^2 H} \sim 5 \times 10^{12}\,{\rm g\,cm^{-3}} \tag{3}$$

2. The midplane pressure follows from vertical hydrostatic balance:

$$-\frac{1}{\rho}\frac{\partial P_{\rm t}}{\partial z} \approx g_{\rm z} \approx \frac{GM_\bullet}{r^2}\frac{z}{r} \rightarrow P_{\rm t} \sim \frac{GM_\bullet \rho_{\rm t} H^2}{R_{\rm t}^3}, \tag{4}$$

where $r \sim R_{\rm t}$ and we have approximated $\partial/\partial z \sim 1/H$ and $z \sim H$.

3. $$c_{\rm s} \approx \left(\frac{P_{\rm t}}{\rho_{\rm t}}\right)^{1/2} \sim H\left(\frac{GM_\bullet}{R_{\rm t}^3}\right)^{1/2} \sim H\Omega_{\rm K}, \tag{5}$$

where $\Omega_{\rm K} \equiv (GM_\bullet/R_{\rm t}^3)^{1/2}$.

4. The accretion (or "viscous") time is

$$t_{\rm acc} \approx \frac{R_{\rm t}^2}{\nu} \sim \frac{R^2}{\alpha \Omega_{\rm K} H^2} \sim 2.5\,{\rm s}, \tag{6}$$

where $\nu = \alpha c_{\rm s} H$.

5. $$\dot{M}_{\rm t} \sim M_{\rm t}/t_{\rm acc} \sim 0.04 M_\odot\,{\rm s^{-1}} \tag{7}$$

6. $$L_{\rm j} = \epsilon_{\rm j}\dot{M}_{\rm t}c^2 \sim 7 \times 10^{50}\,{\rm erg\,s^{-1}} \tag{8}$$

7. $$J_{\rm t} \sim M_{\rm t}(GM_\bullet R_{\rm t})^{1/2} \tag{9}$$

8. If $J_{\rm t} \sim$ constant, then $R_{\rm t} \propto M_{\rm t}^{-2}$, i.e. the disk spreads outwards in time (this is sometimes referred as "viscous spreading").