Binary Evolution Problems Set

PROBLEM 1:
Binary Break-up, Runaway Stars and Supernova Kicks.

Runaway stars are O and B stars which: (i) have large space velocities (up to 200 km/s, compared to 10 or 20 km/s for most stars of this spectral type); and (ii) are always single, whereas most O and B stars are in binary systems.

A possible explanation for their formation is that they were members of a binary system, but that the more massive star (of mass $\mathbf{m_1}$) reached the end of its life, exploded as a supernova and ejected most of its mass; as a result the less massive star (of mass $\mathbf{m_2}$) became unbound, left the binary with its original orbital velocity and now appears as a runaway star.

a) Suppose that the two stars were initially in a circular orbit and that the more massive star instantaneously ejects a mass $\mathbf{\Delta m}$ in the supernova explosion. Assume that the ejection is spherically symmetric in the rest frame of the star. Show that the remnant becomes unbound after the supernova (i.e. show that the total energy of the post-supernova system is positive) if $\mathbf{\Delta m} \geq (\mathbf{m_1} + \mathbf{m_2})/2$.

b) Observations of pulsars have shown that supernova explosions are not perfectly symmetric and that the neutron star receives a recoil kick velocity as a result of the supernova with a typical value of 250 km/s. Explain how, in this case, a binary system can remain bound even if $\mathbf{\Delta m} \geq (\mathbf{m_1} + \mathbf{m_2})/2$ (you may assume that the recoil occurs in a random direction in the frame of the exploding star).

PROBLEM 2: Mass-transfer in binary systems

Part I (Derivation):

Let us consider a binary comprising two stars of mass $\mathbf{M_1}$ and $\mathbf{M_2}$ with an orbital separation $\mathbf{A}$ and orbital period $\mathbf{P}$.

a) Show that the total angular momentum of the binary can be written as:

$$\frac{\mathrm{d}J}{\mathrm{d}t} = -\frac{32}{5}\frac{G^{7/2}}{c^5}\frac{\mu^2 M^{5/2}}{A^{7/2}},$$

where $\mu = M_1M_2 / (M_1+M_2)$ is the reduced mass of the system. Show that for conservative mass transfer (where the total mass and the total angular momentum of the system remains constant), the orbital separation is a minimum when $M_1 = M_2$. [Harder: Sketch the evolution of **A** as a function of time assuming that $M_1 > M_2$ initially and that mass is transferred from star 1 to star 2. How does this behavior of **A** affect the mass-transfer rate, assuming that star 1 attempts to expand at a steady rate?]

b) Even in the absence of mass transfer, the orbit of a binary will shrink due to the emission of gravitational waves, which causes the loss of orbital angular momentum at a rate where $M = M_1 + M_2$ is the total mass of the binary. Show that this implies that the orbital period decreases as:

$$\frac{1}{P}\frac{\mathrm{d}P}{\mathrm{d}t} = -\frac{96}{5}\frac{G^3}{c^5}\frac{M^2\mu}{A^4}.$$

By setting the orbital period decay time (**P/ [dP/dt]**) equal to the age of the Galaxy (~10^{10} yr), determine the maximum separation and hence maximum orbital period for which a binary consisting of (i) two low-mass helium white dwarfs with $M_1 = M_2 = 0.3M_\odot$, (ii) two massive carbon/oxygen white dwarfs with $M_1 = M_2 = 1M_\odot$ and (iii) two neutron stars with $M_1 = M_2 = 1.4M_\odot$ are driven into contact by gravitational wave emission within the age of the Galaxy. Discuss the likely/possible fate of the systems in the three cases.

c) Assume that star 1 loses mass in a stellar wind at a wind mass-loss rate $dM/dt = 10^{-10}M_\odot\ yr^{-1}$ and that the wind is magnetically coupled to the spin of star 1 up to a radius 10**R** away from the star (where **R** is the radius of the star). Assume further that due to the tidal interaction with the companion star, the spin of star 1 is synchronized with the orbital period (i.e. $P_{spin} = P_{orb}$). Estimate the orbital period decay time (**dP/[dP/dt]**) due to this magnetic braking for a system with $M_1 = M_2 = 1M_\odot$ and A = 3R [tip: what is the specific angular momentum loss of the wind?]

Part B (Harder and numerical):

For all projects, first think carefully about what the problem is that you want to solve. Write down the equations that govern the problem and work out which equations you have to solve or integrate. Then start planning the program, again on paper, before you start writing the code. Try to make the plots suitable for scientific publication.

a) Mass stream in the Roche lobe

Calculate the trajectory of a free falling mass stream in the Roche potential starting from L1 (Lagrange Point 1; described in lecture today), falling towards the star at the "center" of the Roche lobe. To do this, integrate the equation of motion of a test particle in the Roche potential. Make plots of the trajectory of the particle for different values of the mass ratio q (at least 5 different values, some above, some below $\mathbf{q = m_1/m_2 = 1}$). Give a short description why this should or should not be a reasonable approximation of the trajectory of a mass stream in a mass-transferring binary.

b) The influence of mass transfer on the binary orbit

Calculate the influence on the orbital separation **a** of mass transfer from star 1 to star 2 in a circular binary for the following cases :

i. For conservative mass transfer ($\mathbf{dm_{tot}/dt} = 0$; $\mathbf{dJ_{tot}/dt} = 0$).

ii. For non-conservative mass transfer in which a fraction $\boldsymbol{\alpha}$ (greek "alpha") of the transferred mass leaves the system with the angular momentum of the accreting star.

iii. For non-conservative mass transfer in which a fraction $\boldsymbol{\beta}$ of the transferred mass leaves the system with the angular momentum of the mass-losing star.

iv. For non-conservative mass transfer in which a fraction $\boldsymbol{\alpha}$ of the transferred mass leaves the system taking a fraction $\boldsymbol{\beta}$ of the specific angular momentum (J_{tot} = m_{tot}) with it to infinity. Calculate the time evolution of the orbit (the actual size of the initial orbit is not important, so take it as 1 in arbitrary units) for two different mass ratios (one larger than 1, one smaller).

Make plots of the change in separation as function of donor mass in units of the total mass for the conservative case and for non-conservative mass transfer with $\boldsymbol{\alpha}$ = 0:1, $\boldsymbol{\alpha}$ = 0:5 and $\boldsymbol{\alpha}$ = 1. Use $\boldsymbol{\beta}$ = 2:5 which corresponds roughly to mass loss from the system at the Lagrange 2 point. Also provide a table with the value of the separation at the moment the donor star has lost half of its mass for all cases and comment on the range of possible outcomes.

c) The evolution of an AM CVn system

AM CVn systems are binaries in which a white dwarf loses mass to another white dwarf. Calculate the evolution of the binary, starting with an 0.2M white dwarf filling its Roche lobe and having an 0.6M white dwarf companion. Assume all the mass is accreted by the accretor, but angular momentum is lost because of gravitational wave radiation.

$$\frac{\dot{J}}{J} = -\frac{32}{5}\frac{G^3}{c^5}\frac{M1M2(M1+M2)}{a^4}$$

You may assume: $R_{\mathrm{WD}} = 0.0123(M_{\mathrm{WD}}/0.6\mathbf{M}_\odot)^{-1/3}\mathbf{R}_\odot$

The problem can be solved assuming $R_{WD} = R_L$ and $dR_{WD}/dt = dR_L/dt$.

[credit: questions modified from stellar evolution courses at Radboud (Nelemans, Verbunt), Cambridge (Part III, Pringle & Tout), Oxford (Podsiadlowski)]