CGWAS continuous wave solutions

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1 Sensitivity of a known pulsar $\mathcal{F}$-statistic search

(a) In MATHEMATICA I would write this:

```
threshold = x /. FindRoot[CDF[ChiSquareDistribution[4], x] ==
0.99, {x, 10}]
```

That is, we find the point at which the CDF (cumulative distribution function) is 0.99 or the "survival function" $1 - \text{CDF}$ is 0.01. The answer is $\boxed{2\mathcal{F} \approx 13.2767}$ or a bit more than 3 times the mean of the distribution. If you don't want to mess with root finders in Python, it's fairly easy to zero in on it by hand.

(b) Now we want to find the noncentrality parameter such that the CDF evaluated at 13.2767 is 0.10. Again in MATHEMATICA:

```
r2 = x /. FindRoot[CDF[NoncentralChiSquareDistribution[4, x],
13.2767] == 0.1, {x, 10}]
```

The answer is $\boxed{2\mathcal{F} \approx 20.7370,}$ or a bit more than 5 times the mean.

(c) The 2/5 comes from averaging a quadrupolar beam pattern over the sky. In the case of F_+ and $F_\times$, it's the quadrupolar beam pattern of the detector. Here "average" also means over inclination and polarization angles of the pulsar's rotation axis. Now imagine you're sitting on the pulsar—it's like averaging over a bunch of locations for LIGO on the neutron star's sky. The emission pattern is quadrupolar, so there's another 2/5. The product of the two 2/5's is 4/25, and following directions at the end gives us $\boxed{11.3845.}$

2 Elastic deformations of neutron stars

(a) In this limit, the crust reacts as if the rest of the neutron star were a point mass located at the center. So $g = GM/r^2$ and $U = 0$. Then we have

$$\frac{8\pi}{15}\sqrt{\frac{32\pi}{15}}48\int_{R-\Delta R}^{R}\frac{\mu r^5\sigma}{GM} \approx \frac{128\pi}{5}\sqrt{\frac{32\pi}{15}}\frac{\mu\sigma R^5\Delta R}{GM}, \tag{1}$$

where we have kept the first term in a power series in $\Delta R/R$.

(b) In this limit, you can use Gauss' Law (for instance) to derive $g = GMr/R^3$, and thus $U = 3$. Then the integral turns into the same thing with 48 replaced by 5, so the number is almost identical for $\Delta R/R = 1/10$.

(c) Plugging in the numbers (you can google things like the solar mass), you should get $\boxed{8 \times 10^{39}\,\mathrm{g\,cm^2}.}$ This is of order 10^{-5} times the fiducial moment of inertia.

3 R-modes

(a) Using the fact that the spherical harmonics are orthonormal, the kinetic energy is just

$$\int d^3r \rho \delta \mathbf{v} \cdot \delta \mathbf{v}^* = \alpha^2 \Omega^2 R^{-2} \int_0^R dr\, \rho r^6. \tag{2}$$

(b) There is no density perturbation, so $\boxed{I^{22} = 0.}$ For the current quadrupole, orthonormality again helps us, and

$$S^{22} = -\frac{32\sqrt{2}\pi}{15} \frac{\alpha\Omega}{Rc} e^{i\omega t} \int_0^R dr\, \rho r^6. \tag{3}$$

(c) Plugging in the expression above for the current quadrupole, we get about $\boxed{3.4 \times 10^{39}\,\mathrm{g\,cm^2}}$ or a bit less than 10^{-5} times the fiducial moment of inertia. (Remember that Ω is an angular frequency.)