1) cosmology preliminaries

→ how to go from theory "of everything" To mathematical metric (a feat!)

→ observe That Universe is homogeneous and isoTropic (implies constant curvature) > 100 Mpc ; assume we are not special

→ introduce family of synchronous observers, yield (why?)

$$ds^2 = -c^2 dt^2 + a^2(t)\left[dx^2 + dy^2 + dz^2\right]$$

FRW/ comoving coordinates

(we're going to assume flatness)

$\boxed{\text{encodes basic fact of expansion of U.}}$

proper distance = a·r

Time as experienced by observers

→ To characterize dynamics, assume perfect-fluid constituents of Universe

$T^{\hat{0}\hat{0}} = \rho \quad T^{\hat{j}\hat{k}} = P\delta^{jk}$ (also non flat)

$\Omega_M \sim 0.27$	cold matter (baryons + CDM)	$P = 0$	$\rho \propto t^{-2}$ $a = t^{2/3}$	$\rho_M = \rho_M^0\left(\frac{a_0}{a}\right)^3$
$\Omega_R \sim 10^{-4}$	radiation (CMB, ν, gravitons)	$P = \frac{\rho}{3}$	$\rho \propto t^2$ $a = t^{1/2}$	$\rho_R = \rho_R^0\left(\frac{a_0}{a}\right)^4$
$\Omega_\Lambda \sim 0.73$	dark energy	$P_\Lambda < -\frac{1}{2}\rho$ ($= -\rho$ for vacuum energy)	$\rho = \text{const}$ $a \propto e^{\pm\sqrt{8\pi G\rho/3}\,t}$	$\rho_\Lambda = \text{const}$ (replenish energy density)

units of $\frac{3}{8\pi}H_0^2 = \rho_c$

EFE + stress-energy cons. $\Rightarrow \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho\left[-\frac{k}{a^2}\right] \quad \dot\rho = -3\frac{\dot a}{a}\left(\rho + \frac{P}{c^2}\right)$

$\frac{d\rho}{da} = -3\frac{\rho + P}{a}$ also non flat radiation dominance to $\frac{a}{a_0} < 10^{-4}$

→ U. expanded at ever decreasing rate from very early time to about now, when transition to dark-energy dominance yields acceleration

- inflation 10^{-36} s to $10^{-33/32}$ (Planck time 10^{-43})
- nucleosynthesis at $z \sim 3\ 10^8$ (few mins) little known before
- radiation to matter dominance at $z \sim 3000$ (70,000 yr)
- recombination (neutral matter → CMB) at $z \sim 1090$ (380,000 yr)
- ~~~~ galaxy formation ($\Delta\rho/\rho \sim 1$) at $z \sim 10$ (2×10^9 yr)

→ cosmological redshift

$$dt_{obs} = \underbrace{\frac{a_0}{a}}_{1+z} dt_{source} \quad \text{@ comoving } r$$

from solving geodesic eq. for photons from periodic source

$$\int_{t_{emit}}^{t_{obs}} \frac{c\,dt}{a(t)} = \int_0^r dr \Rightarrow \frac{\Delta t_{obs}}{a(t_{obs})} = \frac{\Delta t_{emit}}{a(t_{emit})}$$

$$f_{obs} = \frac{f_{source}}{1+z}$$

$$E_{obs} = \frac{E_{source}}{1+z} \quad (\text{from } E = h\nu)$$

use z instead of t, r ~~~~ indexes past light cone

$$L_{obs} = \frac{dE_{obs}}{dt_{obs}} = \frac{(1+z)^{-1} dE_{source}}{(1+z)\,dt_{source}} = (1+z)^{-2} L_{source}$$

from source at distance r and time t / redshift z

$$F_{obs} = \frac{L_{obs}}{4\pi a^2(t_{obs})\, r^2} = \frac{L_{source}}{4\pi a^2(t_{obs}) r^2 (1+z)^2}$$

area over which energy is spread

$d_L = (1+z)\, a(t_0)\, r$ luminosity distance $\qquad 1/H_0 = 4.3$ Gpc

~~~~ $z \simeq \frac{H_0}{c} d_L$ Hubble law with $H_0 = \frac{\dot{a}(t_0)}{a(t_0)} \simeq 70 \frac{km}{s}/Mpc$ the Hubble constant
~~~~

$d_L(z)$ encodes the full expansion history of the Universe

$$r = \int \frac{c\,dt}{a(t)} = \frac{c}{a_0}\int \frac{dz}{H(z)} \Rightarrow d_L = c(1+z)\int \frac{dz}{H(z)}$$

since $dz = \left(\frac{a_0}{a}\right)' dt = -\frac{a_0 \dot{a}}{a^2}\,dt = -\frac{a_0}{a} H\,dt$

$$dt = -\frac{a(t)}{a_0}\frac{dz}{H}$$

$$\left[+\rho_R^0\left(\frac{a_0}{a}\right)^4 +\right]$$

with $H^2 = \frac{8\pi G}{3}\left(\rho_M^0\left(\frac{a_0}{a}\right)^3 + [k\text{ contrib}] + \rho_\Lambda^0\right)$

$$= H_0^2\left(\Omega_M(1+z)^3 + \Omega_\Lambda\right)$$

so $d_L = c(1+z)\frac{1}{H_0}\int \frac{dz}{\sqrt{\Omega_M(1+z)^3 + \Omega_\Lambda}}$ sluzyı tест what does a binary inspiral look like from a cosmological distance?

[introduce signal first from next page]

[what does a photon do?]

in geometrical optics approximation all massless particles follow null geodesics and polarizations don't mix; only redshift occurs

write linearized GW equations on FRW background in terms of conformal time $a(\eta)\,d\eta = dt$

and in terms of conformal metric perturbation $\hat{h}_{ij} = a^{-2} h_{ij}$

yields

$$\nabla^2 \hat{h}_{ij} - \frac{1}{c^2}\frac{\partial^2}{\partial \eta^2}\hat{h}_{ij} - 2\frac{1}{c^2}\frac{1}{a}\frac{da}{d\eta}\frac{\partial}{\partial \eta}\hat{h}_{ij} = \frac{16\pi G}{c^4}\pi^{TT}_{ij}$$

ignore when physical frequency of wave $\gg H$ / converse results in simplification during inflation

leading to solutions in the form

$$h(r,\eta) = \frac{1}{r a(\eta)}\, g(\eta - r/c) = \frac{1}{r a_0}\, g(t - r/c)$$

normalizing η so that it equals t at current epoch

in the exercises

in source frame

$$\begin{cases} h_+ \\ \\ h_\times \end{cases} = \begin{cases} \frac{1+\cos^2\iota}{2} \times \cos \\ \\ \cos\iota \times \sin \end{cases} 2\pi\int dt_s f^s_{gw}(t_s) \times \frac{4}{r}\left(\frac{GM_c}{c^2}\right)^{5/3}\left(\frac{\pi f^S_{gw}(t_s)}{c}\right)^{2/3}$$

replace with $2\pi\int dt_{obs}\, f^{obs}_{gw}(t_{obs})$

replace with $a(t_0)r = d_L/(1+z)$

replace with $(1+z) f^{obs}_{gw}$

$$\frac{4}{d_L}\left(\frac{GM_c(1+z)}{c^2}\right)^{5/3}\left(\frac{\pi f^{obs}_{gw}}{c}\right)^{2/3}$$

and

$$f^{(1)}_{gw}(\tau) = \frac{1}{\pi}\left(\frac{5}{256}\frac{1}{\tau_s}\right)^{3/8}\left(\frac{GM_c}{c^3}\right)^{-5/8}$$

Time to coalescence

replace with $M_c(1+z) \equiv M'$

can measure from change in $f^{(1)}_{gw}$ redshifted chirp mass M'_c

both the GW amplitude and the evolution of freq. w/time are determined by the only timescale — (ν is dimensionless!) in the problem, which is M_c, and which must be redshifted at $t=0$

no hair!

why chirp mass?

$$h \sim G\partial_t^2\, \mu r^2 \sim \omega^2 \underbrace{G^{5/3}\mu M^{2/3}}\,\omega^{-4/3} \sim G^{5/3}$$

$$r^3\omega^2 = GM \to r = G^{1/3}M^{1/3}\omega^{-2/3}$$

has dim $[M]^{5/3}$

set equal to $M_c^{5/3}$

$\to M_c = \mu^{3/5}M^{2/5}$

Thus, measuring h_+ and h_c and f_{obs} yields inclination, d_L, and M'_c

$w = \frac{p_\Lambda}{\rho_\Lambda}$ $\quad w(a) = w_0 + w_a(1-a)$ $\boxed{2.3}$

best: BBO
10^5 NS binaries per year
$h \sim .1\%$
$\Omega_m \sim 1\%$
$w_0 \sim 1\%$, $w_a \sim 10\%$

standard siren strategies

measure D_L and M_c' from waveform and ...

	pros	cons
~~measure~~ z from redshift of host galaxy of em counterpart (e.g., S.H. GRB)	accurate z,	relatively few events ... may not exist for some sources; misidentification
assume small range of M_c from tight prior, infer $1+z = \frac{M_c'}{M_c}$	no counterparts needed!	introduce systematic uncertainty
attribute event to galaxies probabilistically in sky-location box	ditto	more systematics (e.g. rel prob); incompleteness of catalogs; need spectra
use distribution of SNRs	single IFO (no position/inclination)	require joint model of $\frac{dN}{dz\,dM_c}$

other systematics
- strain calibration — few %?
- peculiar velocities — at $z=0.1$: $\delta z = 300\,\frac{\text{km}}{\text{s}}/c$, 1%?; $\delta d_L = \frac{c}{H_0}\delta z = \frac{300\,\frac{\text{km}}{\text{s}}}{70\,\frac{\text{km}}{\text{s}}/\text{Mpc}} = 4\,\text{Mpc}$
- lensing — linear up to 5% error at $z=1$ ← reduce by $\times 2$–3 by considering lensing ~~PDF or by averaging~~ over many signals
- waveform (spins?)

$z = 0.1$, $D_L = 500$ MPc
$z = 0.3$, $D_L = 1500$ MPc

measure **tidal deformability** of NSs

EOS? poor parameter determination

$$\psi^{\text{tidal}}(f) = \frac{3\lambda}{128\eta} \times \frac{x^{5/2}}{M^5} \qquad x = (\pi M f)^{2/3}$$

5 PN formally

$$\lambda = \frac{2}{3} R_{ns}^5 k_2$$

strength of induced quadrupole

k_2: Tidal love number $l=2$

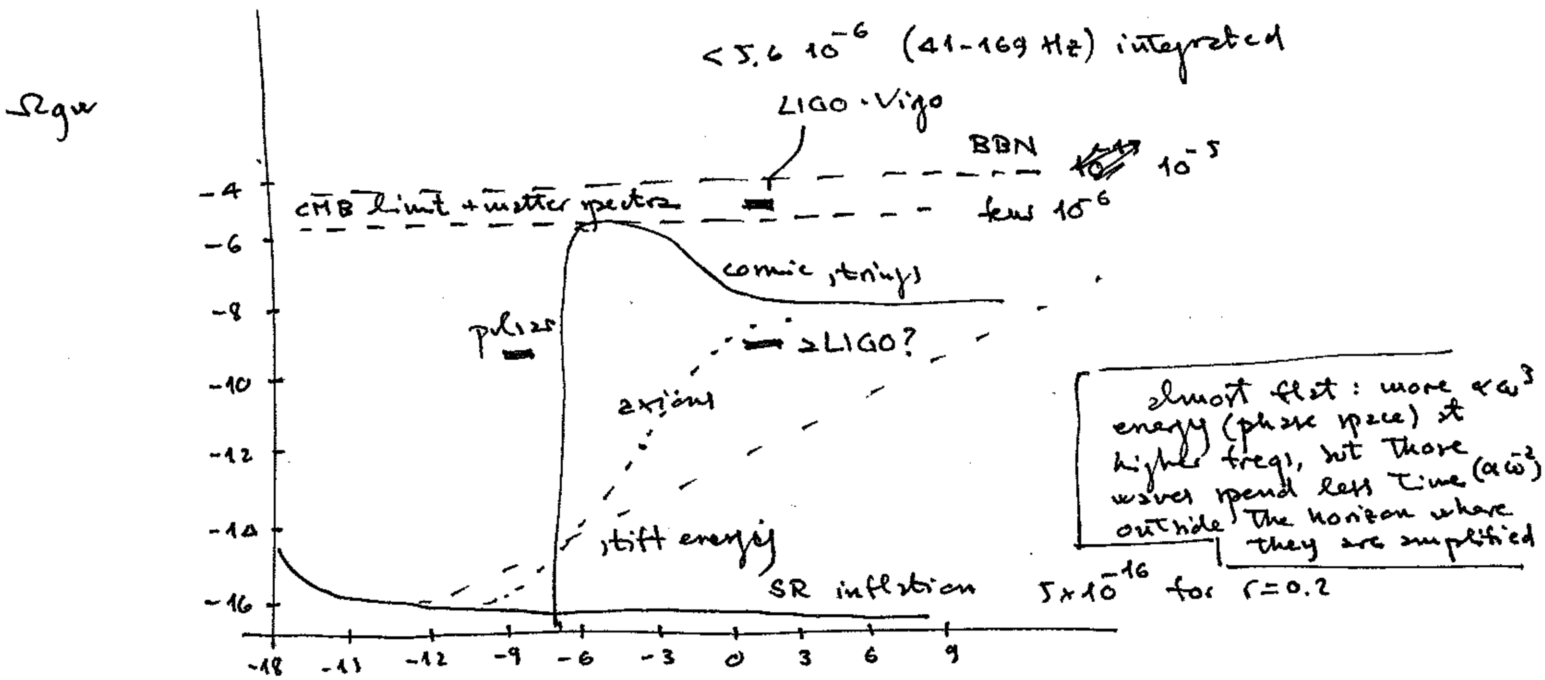
effectively, measure redshifted M and R or measure R/M and infer M

stochastic GWs of cosmological origin

· what are stoch GWs? nonpredictable **unmodeled**, random nature

homogeneous and isotropic

characterized by energy density in GWs (see 8.1)



created by various mechanisms

· amplification of quantum fluctuations due to inflation → 7.1

· first order phase transitions → 7.3

· cosmic superstrings → 7.2

after infl, before BBN (few min $E \sim 3\times10^{8}$)

we don't know what happened before nucleosynthesis

· other nonstandard cosmologies

inflationary GWs

problems in FRW cosmologies

- GUTs (needed at high energy densities) predict magnetic monopoles
- Universe homogeneous to 10^{-5} at photon decoupling (surface of last scattering) which was not in causal contact (horizon problem)
- $|\Omega - 1| \leq 10^{-2}$ now (from CMB) implies $|\Omega - 1| \leq 10^{-62}$ just after Big Bang, since $k \sim a^2 \rho (\Omega - 1)$ (flatness problem)

inflation solves these problems by postulating a 10^{26} exponential increase of the scale factor early on, so that entire observable volume of Universe began as a tiny causally connected region

inflation also creates perturbations of fields by promoting quantum fluctuations to classical

just like a sudden lengthening of a quantum pendulum creates quanta of energy by "mapping" the original ground state into a superposition of different states, so the fast expansion of spacetime creates particles (gravitons!), which are redshifted into classical GWs

$d\rho_{GW}/d\omega$

for simple inflation followed by radiation-dominated FLRW, the number of quanta of freq. ω that are created are

$$N_\omega = \frac{H_0^2 H_{inf}^2}{4\omega^4}$$

constant Hubble parameter during inflation (from exp)

while phase-space density $n_\omega d\omega = \frac{\omega^2}{2\pi^2 c^3} d\omega$

Planck-scale fluctuations in the metric perturbation $\frac{d\rho_{GW}}{d\omega} \sim \frac{\hbar}{c^5}\omega^3$ flat

are amplified by a factor $\beta^2 \sim \left(\frac{H_{inf} H_0}{\omega^2}\right)$ leading to $\frac{d\rho_{GW}}{d\log\omega} = \frac{2\hbar}{c^5} H_{inf}^2 H_0^2$

The flat inflation-produced GW spectrum

$$\Omega_{gw}(f) = \frac{1}{\rho_c}\frac{d\rho_{gw}}{d\ln f} \cong \frac{\hbar G^2}{c^5} f_{inf} \sim \left(\frac{E_{inf}}{E_{planck}}\right)^4$$

$\sqrt{\hbar c^5/G} \sim 2\times10^{19}$ GeV

if inflation occurred at GUT scale (10^{16} GeV)
Then $\Omega_{gw} \sim \cancel{10^{-12}}\ 10^{-9}$

accounting for matter domination yields $\Omega_{gw} \sim \cancel{10^{-9}}\ 10^{-12}$

Ω_{gw} is constant to scale $f_{max} = \frac{a(t_{inf})}{\Delta t}$

Timescale of transition to radiation domination

cosmic strings: symmetry-breaking phase transitions or the ending stages of brane inflation may create cosmic (super)strings, which undergo relativistic oscillations, shrink and ~~&~~ are replaced by pieces of strings larger than the Hubble radius

stochastic backgrounds from first-order phase transitions

first-order → latent energy absorbed into inducing phase changes

water to vapor – nucleated and expanding changes

spontaneous symmetry breaking in early Universe

before symmetry-breaking field in vacuum state...

... after transition, bubbles of true vacuum nucleate

and expand exponentially and eventually collide

gw through three mechanisms:
- colliding bubbles cause anisotropy
- collisions inject energy into matter fields inside bubbles → MHD turbulence
- bubble walls cause charge separation (?) amplified by MHD turbulence

$\Omega \propto f^3$ below f^*
f^{-1} above

$\Omega \propto f^3, f^2$ below f^*
$f^{-5/3}$ above

inv. duration of phase transition

typical peak frequency $f^* \sim 10^{-2}\,\mathrm{mHz}\left(\frac{\beta}{H_*}\right)\left(\frac{kT_*}{100\ \mathrm{GeV}}\right)$

electroweak $kT_* \sim 100$ GeV $\beta/H_* \sim 100$

so $f^* \sim$ mHz (LISA)

but transition not strongly first order

and $\Omega_{gw}(f_*) \sim 10^{-22}$

however, lots of alternative models explored

for LIGO, need PeV

cross-correlation search for stochastic gravitational waves

homogeneous, isotropic stochastic background

$$\frac{d\rho_{gw}}{d\log f} = \rho_{crit}\,\Omega_{gw}(f) \quad \overset{\text{decompose}}{\Rightarrow} \quad \langle \tilde h^*_{+,\times}(f',\hat n')\,\tilde h_{+,\times}(f,\hat n)\rangle =$$

$$= \frac{3\cancel{c}H_0^2}{32\pi^3}\,\frac{\Omega_{gw}(f)}{f^3}\,\delta^2(\hat n,\hat n')\,\delta(f-f')$$

$\rho_{crit} = \frac{3H_0^2}{8\pi G}$

no cross terms

and in terms of interferometer response

$$\langle \tilde h_i^*(f',\hat n'),\,\tilde h_j(f,\hat n)\rangle =$$

$$G_{+,i}(f,\hat n)\,G^*_{+,j}(f',\hat n')\,\langle \tilde h^*_+(f',\hat n')\,\tilde h_+(f,\hat n)\rangle$$

$+$ same for $\times$

and integrating over $\hat n$

cross spectrum

$$S_{ij}(f) = \frac{3H_0^2}{4\pi^2}\,\frac{\Omega_{gw}(f)}{f^3}\,\beta^{-1}\times$$

$$\times\underbrace{\beta\,\frac{1}{4\pi}\int d\hat n\; e^{2\pi i f\hat n\cdot r_{ij}/c}\left[G^*_{+,i}(f,\hat n)\,G_{+,j}(f,\hat n) + G^*_{\times,i}(f,\hat n)\,G_{\times,j}(f,\hat n)\right]}_{\text{overlap reduction function } \gamma_{12}(f)}$$

chosen so that $\gamma_{ii}=1$; $\beta=\frac{5}{2}$ for LIGO-like so $S_{ij}=\frac{3H_0^2}{10\pi^2}\cdots$

e.g. ~ -1 for Hanford and Livingston at $f \ll c/r_{12}$, goes to 0 above 100 Hz

usually $\Omega_{gw}(f) = \Omega_\alpha\,(f/f_{ref})^\alpha$

max efficiency for fixed false alarm

The optimal statistic in the limit of weak background is the weighted cross correlation

$$\mathcal{S} = 4\,\mathrm{Re}\int_0^\infty \frac{\tilde{s}_1^*(f)\,\hat{S}_{12}\,\tilde{s}_2^*(f)}{S_1(f)\,S_2(f)}\,df$$

$S_{12} = \Omega_\alpha \hat{S}_{12}$ so $\mathcal{S} \propto \Omega_\alpha^*$ if signal in data

which results from a multidetector likelihood $\boxed{p(\tilde{s}(f)) \propto \frac{1}{\sqrt{|\Sigma|}}\, e^{-\tilde{s}^*(f)\,\Sigma^{-1}\,\tilde{s}(f)/2}}$

with

$$\Sigma^{(f)} = \begin{pmatrix} S_1^n + S_1^{gw} & S_{12}^{gw} & \cdots \\ S_{21}^{gw} & S_2^n + S_2^{gw} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

can work with this expression $(S_N + S_{GW})^{-1}$

and by approximating Σ^{-1} to $O(\Omega_a^2)$

now $\ln \Lambda \sim \frac{1}{2}\frac{\mathcal{S}^2}{N^2}$ at max Ω_a with $\Omega_{a,est} \sim \frac{\mathcal{S}^2}{N^2}$

a function of $\hat{S}_{12}$, S_1, S_2 and $\tilde{s}_1(f)$, $\tilde{s}_2(f)$

The characteristic SNR ρ for a given S_{12}

is $$\frac{\langle S\rangle}{\sqrt{\mathrm{var}\,S}} = T^{1/2}\sqrt{2\int_0^\infty \frac{S_{12}^2(f)}{S_1(f)\,S_2(f)}\,df} = \rho$$

and it sets the sensitivity of a search. For LIGO S5, S6?

$\rho = 3 \Rightarrow \Omega_0 = 2\times10^{-6}$

upper limits can be derived from the Gaussian statistics of $\mathcal{S}$